

# Thermodynamical Properties and Quasi-localized Energy of the Stringy Dyonic Black Hole Solution

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## ABSTRACT

In this article, we calculate the heat flux passing through the horizon  $\mathbf{TS}|_{r_h}$  and the difference of energy between the Einstein and Møller prescription within the region  $\mathcal{M}$ , in which is the region between outer horizon  $\mathcal{H}_+$  and inner horizon  $\mathcal{H}_-$ , for the modified GHS solution, KLOPP solution and CLH solution. The formula  $E_{\text{Einstein}}|_{\mathcal{M}} = E_{\text{Møller}}|_{\mathcal{M}} - \sum_{\partial\mathcal{M}} \mathbf{TS}$  is obeyed for the mGHS solution and the KLOPP solution, but not for the CLH solution. Also, we suggest a RN-like stringy dyonic black hole solution, which comes from the KLOPP solution under a dual transformation, and its thermodynamical properties are the same as the KLOPP solution.

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# 1 Introduction

Since the discovery of black hole evaporation in 1974 [1], Hawking radiation is an important and outstanding quantum effect arising from the quantization of matter fields in a background space-time with an event horizon. In order to study quantum effects near the event horizon of black hole, a scientific model which unifies quantum mechanics with general relativity must be considered. Until now, superstring theories are still the most interesting candidates for a consistent quantum theory of gravity. From heterotic string theory, the four-dimensional low-energy effective action [2] in which gravity is coupled to electromagnetic and dilaton field is obtained as

$$S = \int d^4x \sqrt{-g} \left[ -R + 2(\nabla\phi)^2 + e^{-2\phi} F^2 \right]. \quad (1)$$

With respect to the  $U(1)$  potential  $A_\mu$ ,  $\phi$  and  $g_{\mu\nu}$ , the field equations of the effective action (1) are

$$\nabla_\mu (e^{-2\phi} F^{\mu\nu}) = 0, \quad (2)$$

$$\nabla^2 \phi + \frac{1}{2} e^{-2\phi} F^2 = 0, \quad (3)$$

$$R_{\mu\nu} = 2\nabla_\mu \phi \nabla_\nu \phi + 2e^{-2\phi} F_{\mu\rho} F_\nu^\rho - \frac{1}{2} g_{\mu\nu} e^{-2\phi} F^2. \quad (4)$$

In 1991, a magnetically charged black hole solution of field equations found by Garfinkle, Horowitz and Strominger (GHS solution) [2] is

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r(r + 2\Theta) d\Omega, \quad (5)$$

$$e^{-2\phi} = e^{-2\phi_0} \left(1 + \frac{2\Theta}{r}\right), \quad (6)$$

$$F = Q \sin\theta d\theta \wedge d\varphi, \quad (7)$$

with mass  $M$ , magnetic charge  $Q$  and dilaton charge  $\Theta = -Q^2 e^{-2\phi_0}/2M$ . Here  $\phi_0$  is the asymptotic constant values of the dilaton. Later, two distinct stringy dyonic black hole solutions are showed by Kallosh *et. al.* [3] using the line element

$$ds^2 = C dt^2 - \frac{dr^2}{C} - D^2 r^2 d\Omega, \quad (8)$$

and by Cheng *et. al.* [4] using the line element

$$ds^2 = A dt^2 - \frac{B}{A} d\bar{r}^2 - \bar{r}^2 d\Omega. \quad (9)$$

The relation between Eq. (8) and (9) is under a radial coordinate transformation

$$r = \int \sqrt{B} d\bar{r}. \quad (10)$$

However, the different choices of coordinates of black hole will present different structures of space-time. Through the article we use Planck units ( $G = c = k_B = \hbar = 1$ ), and follow the convention that Latin indices run from 1 to 3 and Greek indices run from 0 to 3. In this article, we will investigate the thermodynamical properties and the quasi-localized energy complexes of those stringy dyonic black hole solutions.

## 2 Stringy Dyonic Black Hole Solutions

Subsequently, based on Eq. (8), Kallosh, Linde, Ortín, Peet and van Proeyen presented a stringy dyonic black hole solution (KLOPP solution) in the following formulas [3]

$$ds^2 = e^{2U} dt^2 - e^{-2U} dr^{*2} - R^2 d\Omega, \quad (11)$$

$$e^{2\phi} = e^{2\phi_0} \frac{r^* + \Sigma}{r^* - \Sigma}, \quad (12)$$

$$F = \frac{Qe^{\phi_0}}{(r^* - \Sigma)^2} dt \wedge dr^*, \quad (13)$$

$$G = \frac{Pe^{-\phi_0}}{(r^* + \Sigma)^2} dt \wedge dr^*, \quad (14)$$

where

$$e^{2U} = \frac{(r^* - r_+^*)(r^* - r_-^*)}{R^2}, \quad (15)$$

$$R^2 = r^{*2} - \Sigma^2, \quad (16)$$

$$\Sigma = \frac{P^2 - Q^2}{2M}, \quad (17)$$

$$r_{\pm}^* = M \pm \sqrt{M^2 - \Gamma + \Sigma^2}, \quad (18)$$

$$\Gamma = P^2 + Q^2. \quad (19)$$

In particular, the same as GHS solution, the area will go to zero as  $r^* = \pm\Sigma$  causing this surface to be singular. Here, GHS solution would be a special case of KLOPP solution when we introduce a coordinate transformation  $r^* = r + \rho$ , and set electric charge  $Q = 0$  and magnetic charge  $P = Q_m e^{\phi_0}$  in which  $\Sigma = -\Theta$ .

Furthermore, another stringy dyonic black hole solution is found by Cheng, Lin and Hsu (CLH solution) using Eq. (9) in the form [4]

$$ds^2 = \Delta^2 dt^2 - \frac{\sigma^2}{\Delta^2} d\bar{r}^2 - \bar{r}^2 d\Omega, \quad (20)$$

$$e^{2\phi} = e^{2\phi_0} \left( 1 - \frac{2\rho}{\sqrt{\bar{r}^2 + \rho^2}} + \rho \right), \quad (21)$$

$$F_{01} = \frac{Q_e}{\bar{r}^2} e^{2\phi_0}, \quad (22)$$

$$F_{23} = \frac{Q_m}{\bar{r}^2}, \quad (23)$$

where

$$\sigma^2 = \frac{\bar{r}^2}{\bar{r}^2 + \rho^2}, \quad (24)$$

$$\Delta^2 = 1 - \frac{2M}{\bar{r}^2} \sqrt{\bar{r}^2 + \rho^2} + \frac{\beta}{\bar{r}^2}, \quad (25)$$

$$\rho = \frac{1}{2M} (Q_e^2 e^{2\phi_0} - Q_m^2 e^{-2\phi_0}), \quad (26)$$

$$\beta = (Q_e^2 e^{2\phi_0} + Q_m^2 e^{-2\phi_0}). \quad (27)$$

Dissimilarly, CLH solution will not give zero surface area with non-zero radius, and has two event horizons at  $\bar{r}_{\pm} = [(2M^2 - \beta) \pm 2M\sqrt{M^2 - \beta + \rho^2}]^{1/2}$ . At the same time, by using a coordinate transformation  $\bar{r}^2 = r^2 + 2\rho r$ , CLH solution will reduce to GHS solution as setting electric charge  $Q_e = 0$  and magnetic charge  $Q_m = Q$ , in which  $\rho = \Theta$  and  $\beta = -2M\Theta$ .

By those two stringy dyonic black hole solutions, Eqs. (11)-(14) and Eqs. (20)-(23), the GHS solution can be modified to a dyonic black hole solution (mGHS solution) as

$$ds^2 = \Xi dt^2 - \Xi^{-1} dr^2 - \bar{R}^2 d\Omega, \quad (28)$$

where

$$\Xi = \frac{(r - r_+)(r - r_-)}{\bar{R}^2}, \quad (29)$$

$$\bar{R}^2 = r(r + 2\rho), \quad (30)$$

$$r_{\pm} = M - \rho \pm \sqrt{M^2 - \beta + \rho^2}. \quad (31)$$

The parameters  $\beta$  and  $\rho$  of above equation are defined in Eqs. (26)-(27). Note that the modified GHS solution also has two event horizon at  $r_+$  and  $r_-$ .

### 3 Thermodynamics of Stringy Black Hole

In order to investigate the thermodynamical properties of black hole, we need to calculate the temperature  $\mathbf{T}$  and entropy  $\mathbf{S}$  of black hole which are given as

$$\mathbf{T} = \frac{\kappa}{2\pi}, \quad (32)$$

$$\mathbf{S} = \frac{\mathcal{A}}{4}, \quad (33)$$

where the surface gravity  $\kappa$  [5] and area  $\mathcal{A}$  [1] is

$$\kappa = \lim_{r \rightarrow r_H} \left( \frac{1}{2} \frac{\partial_r g_{tt}}{\sqrt{g_{tt} g_{rr}}} \right), \quad (34)$$

$$\mathcal{A} = \int \sqrt{g_{\theta\theta} g_{\phi\phi}} d\theta d\phi. \quad (35)$$

Also, we study the heat flux passing the event horizon of black hole. Hence, the temperature, entropy and heat flux passing the event horizon of foresaid three black hole solution are obtained as :

(i) modified GHS solution

$$\mathbf{T} = \frac{r_h - r_-}{4\pi \bar{R}_h^2} + \frac{r_h - r_+}{4\pi \bar{R}_h^2} - \frac{(r_h + \rho)(r_h - r_+)(r_h - r_-)}{2\pi \bar{R}_h^4}, \quad (36)$$

$$\mathbf{S} = \pi \bar{R}_h^2, \quad (37)$$

$$\mathbf{TS}|_{r_h} = \frac{r_h - r_+}{4} + \frac{r_h - r_-}{4} - \frac{(r_h + \rho)(r_h - r_+)(r_h - r_-)}{2r_h(r_h + 2\rho)}, \quad (38)$$

where  $r_h$  is its event horizon.

(ii) KLOPP solution

$$\mathbf{T} = \frac{r_h^* - r_-^*}{4\pi R_h^2} + \frac{r_h^* - r_+^*}{4\pi R_h^2} - \frac{r_h^*(r_h^* - r_+^*)(r_h^* - r_-^*)}{2\pi R_h^4}, \quad (39)$$

$$\mathbf{S} = \pi R_h^2, \quad (40)$$

$$\mathbf{TS}|_{r_h^*} = \frac{r_h^* - r_+^*}{4} + \frac{r_h^* - r_-^*}{4} - \frac{r_h^*(r_h^* - r_+^*)(r_h^* - r_-^*)}{2(r_h^{*2} - \Sigma^2)}, \quad (41)$$

where  $r_h^*$  means its event horizon.

(iii) CLH solution

$$\mathbf{T} = \frac{M}{2\pi \bar{r}_h^2} + \frac{M\rho^2}{\pi \bar{r}_h^4} - \frac{\beta}{2\pi \bar{r}_h^4} \sqrt{\bar{r}_h^2 + \rho^2}, \quad (42)$$

$$\mathbf{S} = \pi \bar{r}_h^2, \quad (43)$$

$$\mathbf{TS}|_{\bar{r}_h} = \frac{M}{2} + \frac{M\rho^2}{\bar{r}_h^2} - \frac{\beta}{2\bar{r}_h^2} \sqrt{\bar{r}_h^2 + \rho^2}, \quad (44)$$

where  $\bar{r}_h$  typify its event horizon, and satisfy that  $2M\sqrt{\bar{r}_h^2 + \rho^2} = \bar{r}_h^2 + \beta$ . So, the temperature and heat flux can be replaced as

$$\mathbf{T} = \frac{M}{2\pi\bar{r}_h^2} + \frac{M\rho^2}{\pi\bar{r}_h^4} - \frac{\beta}{4\pi M\bar{r}_h^2} - \frac{\beta^2}{4\pi M\bar{r}_h^4}, \quad (45)$$

$$\mathbf{TS}|_{\bar{r}_h} = \frac{M}{2} + \frac{M\rho^2}{\bar{r}_h^2} - \frac{\beta}{4M} - \frac{\beta^2}{4M\bar{r}_h^2}. \quad (46)$$

These results of KLOPP solution are very similar to GHS solution, but CLH solution not.

## 4 Four Kinds of Energy Complexes

Next, let us to consider the energy complex in the Einstein [6], Weinberg [7], Landau-Lifshitz [8] and Møller[9] prescription. The energy component of the Einstein energy-momentum complex [6] is given by

$$E_{\text{Einstein}} = \frac{1}{16\pi} \int \frac{\partial H_0^{0i}}{\partial x^i} d^3x, \quad (47)$$

where

$$H_0^{0i} = \frac{g_{00}}{\sqrt{-g}} \frac{\partial}{\partial x^m} [(-g)g^{00}g^{im}]. \quad (48)$$

Applying Gauss's theorem one obtain

$$E_{\text{Einstein}} = \frac{1}{16\pi} \oint H_0^{0i} \hat{n}_i \cdot d\vec{S}, \quad (49)$$

where  $\hat{n}_i$  is the outward unit normal vector over the infinitesimal surface element  $d\vec{S}$ . In the same way, the energy component of the Weinberg energy-momentum complex [7] within radius  $r$  is given by

$$E_{\text{Weinberg}} = \frac{1}{16\pi} \oint Q^{i00} \hat{n}_i \cdot d\vec{S}, \quad (50)$$

where

$$Q^{i00} = \frac{\partial h_{jj}}{\partial x^i} - \frac{\partial h_{ij}}{\partial x^j}, \quad (51)$$

and of the Landau-Lifshitz energy-momentum complex [8]

$$E_{\text{Landau-Lifshitz}} = \frac{1}{16\pi} \oint \frac{\partial \lambda^{00ik}}{\partial x^k} \hat{n}_i \cdot d\vec{S}, \quad (52)$$

where

$$\lambda^{00ik} = (-g) (g^{00}g^{ik}). \quad (53)$$

According to the definition of the Møller energy-momentum complex [9] and Gauss's theorem, the energy component is given as

$$E_{\text{Møller}} = \frac{1}{8\pi} \oint \chi_0^{0i} \hat{n}_i \cdot d\vec{S}, \quad (54)$$

where

$$\chi_0^{0i} = \sqrt{-g} \left( -\frac{\partial g_{00}}{\partial x^i} \right) g^{00} g^{ii}. \quad (55)$$

Beginning with the modified GHS solution and the KLOPP solution, their line element is like Eq. (8) as

$$ds^2 = C dt^2 - \frac{dr^2}{C} - Dr^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (56)$$

The energy component within radius  $r$  obtained by the Einstein complex is

$$E_{\text{Einstein}} = \frac{r}{2}(1 - CD) - \frac{r^2 C}{2} \frac{dD}{dr}, \quad (57)$$

by the Weinberg complex is

$$E_{\text{Weinberg}} = \frac{r}{2} \left( \frac{1}{C} - D \right) - \frac{r^2}{2} \frac{dD}{dr}, \quad (58)$$

and by the Landau-Lifshitz energy complex is

$$E_{\text{Landau-Lifshitz}} = \frac{rD}{2} \left( \frac{1}{C} - D \right) - \frac{r^2 D}{2} \frac{dD}{dr}. \quad (59)$$

For the modified GHS solution, the relation of energy component of the Einstein, Weinberg and Landau-Lifshitz complex is

$$E_{\text{Einstein}}^{(\text{mGHS})} = \Xi E_{\text{Weinberg}}^{(\text{mGHS})} = \frac{r^2 \Xi}{\bar{R}^2} E_{\text{Landau-Lifshitz}}^{(\text{mGHS})}, \quad (60)$$

and in the Einstein prescription, the expression of energy within radius  $r$  is

$$E_{\text{Einstein}}^{(\text{mGHS})} = \frac{r_+ + r_-}{2} - \frac{r_+ r_-}{2r} + \frac{\rho(r - r_+)(r - r_-)}{2\bar{R}^2}. \quad (61)$$

Similarly, for the KLOPP solution, the relation between these three energy component is

$$E_{\text{Einstein}}^{(\text{KLOPP})} = e^{2U} E_{\text{Weinberg}}^{(\text{KLOPP})} = \frac{r^{*2} e^{2U}}{R^2} E_{\text{Landau-Lifshitz}}^{(\text{KLOPP})}, \quad (62)$$

and the expression of energy in the Einstein prescription is

$$E_{\text{Einstein}}^{(\text{KLOPP})} = \frac{r_+^* + r_-^*}{2} - \frac{r_+^* r_-^*}{2r^*} - \frac{\Sigma^2(r^* - r_+^*)(r^* - r_-^*)}{r^* R^2}. \quad (63)$$

On the other hand, the energy component with radius  $r$  obtained using the Møller complex is

$$E_{\text{Møller}} = \frac{r^2}{2} D \frac{dC}{dr}, \quad (64)$$

and its energy values of the modified GHS solution and the KLOPP solution are

$$E_{\text{Møller}}^{(\text{mGHS})} = \frac{r - r_+}{2} + \frac{r - r_-}{2} - \frac{(r + \rho)(r - r_+)(r - r_-)}{R^2}, \quad (65)$$

$$E_{\text{Møller}}^{(\text{KLOPP})} = \frac{r^* - r_+^*}{2} + \frac{r^* - r_-^*}{2} - \frac{r^*(r^* - r_+^*)(r^* - r_-^*)}{R^2}. \quad (66)$$

In the case of the CLH solution, its line element is given as Eq. (9)

$$ds^2 = A dt^2 - \frac{B}{A} d\bar{r}^2 - \bar{r}^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (67)$$

Thus, the energy component within radius  $\bar{r}$  obtained by the Einstein complex is

$$E_{\text{Einstein}} = \frac{\bar{r}A}{2\sqrt{B}} \left( \frac{B}{A} - 1 \right), \quad (68)$$

by the Weinberg complex is

$$E_{\text{Weinberg}} = \frac{\bar{r}}{2} \left( \frac{B}{A} - 1 \right), \quad (69)$$

by the Landau-Lifshitz complex is

$$E_{\text{Landau-Lifshitz}} = \frac{\bar{r}}{2} \left( \frac{B}{A} - 1 \right), \quad (70)$$

and by the Møller complex is

$$E_{\text{Møller}} = \frac{\bar{r}^2}{2\sqrt{B}} \frac{dA}{d\bar{r}}. \quad (71)$$

The relation of energy component of the Einstein, Weinberg and Landau-Lifshitz complex is

$$E_{\text{Einstein}}^{(\text{CLH})} = \frac{\Delta^2}{\sigma} E_{\text{Weinberg}}^{(\text{CLH})} = \frac{\Delta^2}{\sigma} E_{\text{Landau-Lifshitz}}^{(\text{CLH})}. \quad (72)$$

In previous studies of Yang *et. al.* [10], the expression of Einstein's energy complex within radius  $\bar{r}$  is

$$E_{\text{Einstein}}^{(\text{CLH})} = M + \frac{M\rho^2}{\bar{r}^2} - \frac{\beta\sqrt{\bar{r}^2 + \rho^2}}{2\bar{r}^2} - \frac{\rho^2}{2\sqrt{\bar{r}^2 + \rho^2}}, \quad (73)$$

and of Møller's energy complex is

$$E_{\text{Møller}}^{(\text{CLH})} = M + \frac{2M\rho^2}{\bar{r}^2} - \frac{\beta\sqrt{\bar{r}^2 + \rho^2}}{\bar{r}^2}. \quad (74)$$



## 5 Quasi-localized Energy Complexes and Thermodynamical Potentials

Moreover, the difference of energy between the Einstein and Møller prescription [11] is defined as

$$\Delta E = E_{\text{Einstein}} - E_{\text{Møller}}, \quad (75)$$

and its values for the mGHS, KLOPP and CLH solution are

$$\Delta E^{(\text{mGHS})} = (r_+ + r_-) - \left( r + \frac{r_+ r_-}{2r} \right) \quad (76)$$

$$+ \frac{(r - r_+)(r - r_-)}{R^2} (r + 2\rho), \quad (77)$$

$$\Delta E^{(\text{KLOPP})} = (r_+^* + r_-^*) - \left( r^* + \frac{r_+^* r_-^*}{2r^*} \right) \quad (78)$$

$$+ \frac{(r^* - r_+^*)(r^* - r_-^*)}{R^2} \left( r^* - \frac{\Sigma^2}{r^*} \right), \quad (79)$$

$$\Delta E^{(\text{CLH})} = \frac{\beta \sqrt{\bar{r}^2 + \rho^2}}{2\bar{r}^2} - \frac{M\rho^2}{\bar{r}^2} - \frac{\rho^2}{2\sqrt{\bar{r}^2 + \rho^2}}. \quad (80)$$

In the mGHS, KLOPP and CLH solutions, they are the existence of two different Cauchy horizons, event horizon  $\mathcal{H}_+$  is located at  $r = r_+$  and inner horizon  $\mathcal{H}_-$  is located at  $r = r_-$ . So, we can suppose that  $\mathcal{M} = \{(t, r, \theta, \phi) | r_+ > r > r_-\}$  is the region between  $\mathcal{H}_+$  and  $\mathcal{H}_-$ . However, one suggest that the temperature of the inner Cauchy horizon must be defined as [12]

$$\mathbf{T}|_{r_-} \equiv - \frac{\kappa}{2\pi} \Big|_{r_-}. \quad (81)$$

Because of

$$\Delta E^{(\text{mGHS})} \Big|_{r_-}^+ = - \left( \frac{r_+ - r_-}{2} \right), \quad (82)$$

$$\mathbf{TS}|_{r_+} = \frac{r_+ - r_-}{4} = \mathbf{TS}|_{r_-}, \quad (83)$$

for the mGHS solution, we have

$$E_{\text{Einstein}}^{(\text{mGHS})} \Big|_{\mathcal{M}} = E_{\text{Møller}}^{(\text{mGHS})} \Big|_{\mathcal{M}} - \sum_{\partial\mathcal{M}} \mathbf{TS}. \quad (84)$$

Similarly, base on these results

$$\Delta E^{(\text{KLOPP})} \Big|_{r_-}^{r_+^*} = - \left( \frac{r_+^* - r_-^*}{2} \right), \quad (85)$$

$$\mathbf{TS}|_{r_+^*} = \frac{r_+^* - r_-^*}{4} = \mathbf{TS}|_{r_-^*}, \quad (86)$$

we also can obtain

$$E_{\text{Einstein}}^{(\text{KLOPP})}|_{\mathcal{M}} = E_{\text{Møller}}^{(\text{KLOPP})}|_{\mathcal{M}} - \sum_{\partial\mathcal{M}} \mathbf{TS}. \quad (87)$$

Base on the viewpoint of Nester *et. al.* [13], we could suggest  $E_{\text{Einstein}}$  and  $E_{\text{Møller}}$  as thermodynamic potentials, because Eq. (84) and Eq. (87) are like the Legendre transformation. Furthermore, for the CLH solution, the heat fluxes on those two Cauchy horizons are

$$\mathbf{TS}|_{\bar{r}_+} = \frac{\bar{r}_+^2}{4M}, \quad (88)$$

$$\mathbf{TS}|_{\bar{r}_-} = -\frac{\bar{r}_-^2}{4M}, \quad (89)$$

and the difference of energy in the region  $\mathcal{M}$  is

$$\Delta E^{(\text{CLH})}|_{\bar{r}_-}^{\bar{r}_+} = \frac{\beta(\bar{r}_-^2 - \bar{r}_+^2)}{4M\beta + 4M\rho^2}, \quad (90)$$

Here, we will obtain that

$$E_{\text{Einstein}}^{(\text{CLH})}|_{\mathcal{M}} = E_{\text{Møller}}^{(\text{CLH})}|_{\mathcal{M}} - \frac{\beta}{\beta + \rho^2} \sum_{\partial\mathcal{M}} \mathbf{TS}, \quad (91)$$

and can not get the same result, like Eq. (83) and (86), except  $\rho = 0$ .

## 6 Conclusions

In this article, we have calculated two kinds of physical properties, thermodynamical and mechanical properties, and have obtained the results of both properties about those three stringy black hole solutions. The thermodynamical properties include the temperature  $\mathbf{T}$ , the entropy  $\mathbf{S}$  and the heat flux passing through the horizon  $\mathbf{TS}|_{r_h}$ , and the mechanical properties include four kinds of energy complexes  $E_{\text{Einstein}}$ ,  $E_{\text{Weinberg}}$ ,  $E_{\text{Landau-Lifshitz}}$  and  $E_{\text{Møller}}$ , and the difference of energy between the Einstein and Møller prescription  $\Delta E$ . For mGHS solution and KLOPP solution, particularly, the difference of energy between the Einstein and Møller prescription within the region  $\mathcal{M}$  is equal to the heat flux which is exhibited on every boundary of  $\mathcal{M}$ , like as the formula that we have pointed out [14]

$$E_{\text{Einstein}}|_{\mathcal{M}} = E_{\text{Møller}}|_{\mathcal{M}} - \sum_{\partial\mathcal{M}} \mathbf{TS}, \quad (92)$$

and the heat flux passing by the outer horizon  $\mathcal{H}_+$  and by inner horizon  $\mathcal{H}_-$  are the same, like as

$$\mathbf{TS}|_{\mathcal{H}_+} = \mathbf{TS}|_{\mathcal{H}_-}. \quad (93)$$

However, on the case of CLH solution, because of

$$\mathbf{TS}|_{\bar{r}_+} \neq \mathbf{TS}|_{\bar{r}_-}, \quad (94)$$

we would not obtain the same identical equation (92). The relation of coordinate transformation between the mGHS solution and KLOPP solution is a linear transformation  $r^* = r + \rho$ , but between the KLOPP solution and CLH solution is non-linear transformation  $r^{*2} = \bar{r}^2 + \rho^2$ , in which the relation between  $r^*$  and  $\bar{r}$  is like the hyperbolic functions  $\cosh^2 x - \sinh^2 x = 1$ . Thus, the space-time structure of KLOPP solution is homeomorphism with mGHS solution, but not with CLH solution. According to this condition (93), there occurs equal heat fluxes on both Cauchy horizons of mGHS solution and KLOPP solution. Then, mGHS solution or KLOPP solution is more suitable to study the thermodynamics of stringy black hole solution than CLH solution.

On the other hand, the effective action Eq. (1) and field equation Eq. (3) and Eq. (4) will be invariant under [2]

$$F_{\mu\nu} \longrightarrow \tilde{F}_{\mu\nu} = \frac{1}{2} e^{-2\phi} \epsilon_{\mu\nu}^{\lambda\eta} F_{\lambda\eta}. \quad (95)$$

Let us introduce the notation

$$\bar{Q} = \frac{1}{\sqrt{2}} (Q + P), \quad (96)$$

$$\bar{P} = \frac{1}{\sqrt{2}} (Q - P), \quad (97)$$

so that

$$\Sigma = -\frac{\bar{Q}\bar{P}}{M}, \quad (98)$$

$$\beta = \Gamma = \bar{Q}^2 + \bar{P}^2. \quad (99)$$

The Eq. (15) in the KLOPP solution will be modified as

$$e^{2U} = \left( 1 - \frac{2M}{r^*} + \frac{\bar{Q}^2 + \bar{P}^2 - \Sigma^2}{r^{*2}} \right) \frac{r^{*2}}{R^2}, \quad (100)$$

This new solution is RN-like black hole solution and its thermal properties is the same as KLOPP solution.

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